Bài tập phương trình lượng giác ôn thi đại học

Giải các phương trình sau

1) $2\sqrt{2}\cos 2x+\sin 2x\cos\left(x+\frac{3\pi}{4}\right)-4\sin\left(x+\frac{\pi}{4}\right)=0$

$2\sqrt{2}\cos 2x+\sin 2x\cos(x+\frac{3\pi}{4})-4\sin(x+\frac{\pi}{4})=0 \Leftrightarrow$

$2\sqrt{2}\cos 2x+\sin 2x(\cos x.\cos\frac{3\pi}{4}-\sin x\sin\frac{3\pi}{4})-4(\sin x\cos\frac{\pi}{4}+\cos x\sin\frac{\pi}{4})=0$

$\Leftrightarrow$ 4cos2x-sin2x(sinx+cosx)-4(sinx+cosx)=0 $\Leftrightarrow$ (sinx+cosx)[4(cosx-sinx)-sin2x-4]=0

(3) : Đặt $t=\sin x-\cos x=\sqrt{2}\sin(x-\frac{\pi}{4})$, Điều kiện $|t|\le\sqrt{2}$ (*) $\Rightarrow \sin 2x=1-t^2$, thay vào (2) được PT: $t^2-4t-5=0$ $\Leftrightarrow$ t =-1(t/m (*)) hoặc t =5(loại)

Với t=-1 ta tìm được nghiệm x là : $x=k2\pi$ hoặc x=$\frac{3\pi}{2}+k2\pi$.

KL: Họ nghiệm của hệ PT là: $x=-\frac{\pi}{4}+k\pi$, $x=k2\pi$ và x=$\frac{3\pi}{2}+k2\pi$

Giải

$\Leftrightarrow\left[\begin{array}{ll}\sin x+\cos x=0 & (2)\\ 4(\cos x-\sin x)-\sin 2x-4=0 & (3)\end{array}\right.$. PT (2) có nghiệm $x=-\frac{\pi}{4}+k\pi$.

Với t =-1 ta tìm được nghiệm x là : $x=k2\pi$ hoặc x=$\frac{3\pi}{2}+k2\pi$.

KL: Họ nghiệm của hệ PT là: $x=-\frac{\pi}{4}+k\pi$, $x=k2\pi$ và x=$\frac{3\pi}{2}+k2\pi$

2) Giải phương trình $\sin^2 x+\sin x.\cos 3x+\cos^2 3x=\frac{3}{4}$

$$pt\Leftrightarrow\left(\sin x+\frac{1}{2}\cos 3x\right)^2+\frac{3}{4}\cos^2 3x=\frac{3}{4}$$

$$\Leftrightarrow\left(\sin x+\frac{1}{2}\cos 3x\right)^2=\frac{3}{4}\sin^2 3x\Leftrightarrow\left[\begin{array}{l}\left(\sin x+\frac{1}{2}\cos 3x\right)=\frac{\sqrt{3}}{2}\sin 3x\\ \left(\sin x+\frac{1}{2}\cos 3x\right)=-\frac{\sqrt{3}}{2}\sin 3x\end{array}\right.$$

$$\Leftrightarrow \left[\begin{array}{l} \frac{1}{2}\cos 3x - \frac{\sqrt{3}}{2}\sin 3x = -\sin x \\ \frac{1}{2}\cos 3x + \frac{\sqrt{3}}{2}\sin 3x = -\sin x \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \sin\left(\frac{\pi}{6} - 3x\right) = \sin(-x) \\ \sin\left(\frac{\pi}{6} + 3x\right) = \sin(-x) \end{array}\right.$$

$$\Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{12} - k\pi; x = \frac{-5\pi}{12} - k\pi \\ x = \frac{-\pi}{24} + \frac{k\pi}{2}; x = \frac{5\pi}{12} + k\pi \end{array}\right.$$

3) $\cos 2x - \tan^2 x = \dfrac{\cos^2 x + \cos^3 x - 1}{\cos^2 x}$

ĐK cosx $\neq 0$, pt được đưa về

$$\cos 2x - \tan^2 x = 1 + \cos x - (1 + \tan^2 x) \Leftrightarrow 2\cos^2 x - \cos x - 1 = 0$$

Giải tiếp được cosx = 1 và cosx = 0,5 rồi đối chiếu đk để đưa ra ĐS:

$$x = k2\pi, x = \pm\frac{2\pi}{3} + k2\pi; \text{ hay } x = k\frac{2\pi}{3}.$$

4) $2\cos 3x(2\cos 2x + 1) = 1$

Nhận xét $x = k\pi, k \in Z$ không là nghiệm của phương trình đã cho nên ta có:

$2\cos 3x(3 - 4\sin^2 x) = 1 \Leftrightarrow 2\cos 3x(3\sin x - 4\sin^3 x) = \sin x$

$\Leftrightarrow 2\cos 3x\sin 3x = \sin x \Leftrightarrow \sin 6x = \sin x$

$$\Leftrightarrow \left[\begin{array}{l} 6x = x + m2\pi \\ 6x = \pi - x + m2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \frac{2m\pi}{5} \\ x = \frac{\pi}{7} + \frac{2m\pi}{7} \end{array}\right. ; m \in Z$$

5) : 2(tanx – sinx) + 3(cotx – cosx) + 5 = 0

Phương trình đã cho tương đương với :

2(tanx + 1 – sinx) + 3(cotx + 1 – cosx) = 0

$$\Leftrightarrow 2\left(\frac{\sin x}{\cos x} + 1 - \sin x\right) + \left(\frac{\cos x}{\sin x} + 1 - \cos x\right) = 0$$

$$\Leftrightarrow \frac{2(\sin x + \cos x - \cos x.\sin x)}{\cos x} + \frac{3(\sin x + \cos x - \cos x.\sin x)}{\sin x} = 0$$

$$\Leftrightarrow \left(\frac{2}{\cos x} + \frac{3}{\sin x}\right)(\cos x + \sin x - \cos x.\sin x) = 0$$

- Xét $\dfrac{2}{\cos x} + \dfrac{3}{\sin x} = 0 \Leftrightarrow \tan x = \dfrac{-3}{2} = \tan\alpha \Leftrightarrow x = \alpha + k\pi$
- Xét : sinx + cosx – sinx.cosx = 0 . Đặt t = sinx + cosx

với $t \in \left[-\sqrt{2}; \sqrt{2}\right]$. Khi đó phương trình trở thành:

$t-\frac{t^2-1}{2}=0 \Leftrightarrow t^2-2t-1=0 \Leftrightarrow t=1-\sqrt{2}$

Suy ra : $\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)=1-\sqrt{2} \Leftrightarrow \cos\left(x-\frac{\pi}{4}\right)=\frac{1-\sqrt{2}}{\sqrt{2}}=\cos\beta$

6) $2\sin\left(2x-\frac{\pi}{6}\right)+4\sin x+1=0.$

Ta có : $2\sin\left(2x-\frac{\pi}{6}\right)+4\sin x+1=0.$

$\Leftrightarrow \sqrt{3}\sin 2x - \cos 2x + 4\sin x + 1 = 0$

$\Leftrightarrow \sqrt{3}\sin 2x + 2\sin^2 x + 4\sin x = 0$

$\Leftrightarrow \sin x\,(\sqrt{3}\cos x + \sin x + 2) = 0$

$\Leftrightarrow$ $\sin x = 0$ (1) hoặc $\sqrt{3}\cos x + \sin x + 2 = 0$ (2)

+ (1) $\Leftrightarrow x=k\pi$

+ (2) $\Leftrightarrow \frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x=-1$

$\Leftrightarrow \sin\left(x+\frac{\pi}{3}\right)=-1 \Leftrightarrow x=-\frac{5\pi}{6}+k2\pi$

7) $\sin^3 x+\cos^3 x=\cos 2x.(2\cos x-\sin x)$ KQ: $\left[\begin{array}{l} x=\frac{\pi}{2}+k\pi \\ x=-\frac{\pi}{4}+l\pi \\ x=\arctan\frac{1}{2}+m\pi \end{array}\right.$ $(k,l,m\in\mathbb{Z})$

8) $\sin 2x(\cos x+3)-2\sqrt{3}\cos^3 x-3\sqrt{3}\cos 2x+8\left(\sqrt{3}\cos x-\sin x\right)-3\sqrt{3}=0$

$\Leftrightarrow (\sqrt{3}\cos x-\sin x)(-2\cos^2 x-6\cos x+8)=0$

$\Leftrightarrow \left[\begin{array}{l} \sqrt{3}\cos x-\sin x=0 \\ \cos^2 x+3\cos x-4=0 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \tan x=\sqrt{3} \\ \cos x=1 \\ \cos x=4 (loai) \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{3}+k\pi \\ x=k2\pi \end{array}\right., k\in Z$

9) $9\sin x + 6\cos x - 3\sin 2x + \cos 2x = 8$

Phương trình đã cho tương đương với

$9\sin x + 6\cos x - 6\sin x.\cos x + 1 - 2\sin^2 x = 8$

$\Leftrightarrow 6\cos x(1 - \sin x) - (2\sin^2 x - 9\sin x + 7) = 0$

$\Leftrightarrow 6\cos x(1 - \sin x) - (\sin x - 1)(2\sin x - 7) = 0$

$\Leftrightarrow (1-\sin x)(6\cos x + 2\sin x - 7) = 0$

$\Leftrightarrow \left[\begin{array}{l} 1-\sin x=0 \\ 6\cos x+2\sin x-7=0\ (VN) \end{array}\right.$

$x=\frac{\pi}{2}+k2\pi$

10) $\dfrac{\sqrt{3}}{\cos^2 x}+\dfrac{4+2\sin 2x}{\sin 2x}-2\sqrt{3}=2(\text{cotg}\,x+1)$

Phương trình đã cho tương đương với:

$$\sqrt{3}\left(1+\text{tg}^2x\right)+\frac{4}{\sin 2x}-2\sqrt{3}=2\text{cotg}\,x$$

$$\Leftrightarrow \sqrt{3}\text{tg}^2x+\frac{2(\sin^2 x+\cos^2 x)}{\sin x\cos x}-\sqrt{3}=2\text{cotg}\,x$$

$$\Leftrightarrow \sqrt{3}\text{tg}^2x+2\text{tg}\,x-\sqrt{3}=0$$

$$\left[\begin{array}{l}\text{tg}\,x=-\sqrt{3}\\ \text{tg}\,x=\dfrac{1}{\sqrt{3}}\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=-\dfrac{\pi}{3}+k\pi\\ x=\dfrac{\pi}{6}+k\pi\end{array}\right.$$

KL: So sánh với điều kiện phương trình có nghiệm : $x=\dfrac{\pi}{6}+k\dfrac{\pi}{2}$; $k\in\boldsymbol{Z}$

11) $\sin 4x+\cos 4x=4\sqrt{2}\sin(x+\dfrac{\pi}{4})-1$

$\Leftrightarrow$ (cos x + sin x) (cos x – sin x) (sin 2x + cos 2x) = 2(sin x + cos x)

$$\Leftrightarrow \left[\begin{array}{l}\sin x+\cos x=0\\ (\cos x-\sin x)(\sin 2x+\cos 2x)=2\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=-\dfrac{\pi}{4}+k\pi\\ \cos 3x-\sin x=2\end{array}\right.$$

Chứng minh được phương trình cos 3x – sin x = 2 vô nghiệm

KL: x = $-\dfrac{\pi}{4}+k\pi$

12) $2\cos 6x+2\cos 4x-\sqrt{3}\cos 2x=\sin 2x+\sqrt{3}$

$4\cos 5x\cos x = 2\sin x\cos x + 2\sqrt{3}\cos^2 x \Leftrightarrow \left[\begin{array}{l}\cos x=0\\ 2\cos 5x=\sin x+\sqrt{3}\cos x\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\cos x=0\\ \cos 5x=\cos(x-\dfrac{\pi}{6})\end{array}\right.$

$$\Leftrightarrow \left[\begin{array}{l}x=\dfrac{\pi}{2}+k\pi\\ x=-\dfrac{\pi}{24}+\dfrac{k\pi}{2}\\ x=\dfrac{\pi}{42}+\dfrac{k2\pi}{7}\end{array}\right.$$

13) $\dfrac{4\cos 3x\cos x-2\cos 4x-4\cos x+\tan\dfrac{x}{2}\tan x+2}{2\sin x-\sqrt{3}}=0$

Điều kiện: $\sin x \neq \frac{\sqrt{3}}{2}$ và $\cos\frac{x}{2} \neq 0$ và $\cos x \neq 0$

Biến đổi pt về: $4\cos^3x - 4\cos^2x - \cos x + 1 = 0 \Leftrightarrow \left[\begin{array}{l}\cos x = 1\\ \cos x = \pm\frac{1}{2}\end{array}\right.$

14) $1 + \sqrt{3}(\sin x + \cos x) + \sin 2x + \cos 2x = 0$

Phương trình đã cho tương đương với $(\sqrt{3}\sin x + \sin 2x) + \left[\sqrt{3}\cos x + (1+\cos 2x)\right] = 0$

$\Leftrightarrow (\sqrt{3}\sin x + 2\sin x.\cos x) + (\sqrt{3}\cos x + 2\cos^2 x) = 0 \Leftrightarrow \sin x(\sqrt{3} + 2\cos x) + \cos x(\sqrt{3} + 2\cos x) = 0$

$\Leftrightarrow (\sqrt{3} + 2\cos x)(\sin x + \cos x) = 0 \Leftrightarrow \left[\begin{array}{l}\cos x = -\frac{\sqrt{3}}{2}\\ \sin x = -\cos x\end{array}\right.$

$\Leftrightarrow \left[\begin{array}{l}x = \pm\frac{5\pi}{6} + k2\pi\\ \tan x = -1\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x = \pm\frac{5\pi}{6} + k2\pi\\ x = -\frac{\pi}{4} + k\pi\end{array}\right. , k \in Z$

15) $4\sin^3 x.\cos 3x + 4\cos^3 x.\sin 3x + 3\sqrt{3}\cos 4x = 3$

Phương trình đã cho tương đương với phương trình :

1. Phương trình : $4\sin^3 x.\cos 3x + 4\cos^3 x.\sin 3x + 3\sqrt{3}\cos 4x = 3$

$\Leftrightarrow 4[(1-\cos^2 x)\sin x.\cos 3x + (1-\sin^2 x)\cos x.\sin 3x] + 3\sqrt{3}\cos 4x = 3$

$\Leftrightarrow 4[(\sin x.\cos 3x + \cos x.\sin 3x) - \cos x\sin x(\cos x.\cos 3x + \sin x.\sin 3x)] + 3\sqrt{3}\cos 4x = 3$

$\Leftrightarrow 4[\sin 4x - \frac{1}{2}\sin 2x.\cos 2x] + 3\sqrt{3}\cos 4x = 3 \Leftrightarrow 4\left(\sin 4x - \frac{1}{4}\sin 4x\right) + 3\sqrt{3}\cos 4x = 3 \Leftrightarrow 3\sin 4x + 3\sqrt{3}\cos 4x = 3$

$\Leftrightarrow \sin 4x + \sqrt{3}\cos 4x = 1 \Leftrightarrow \frac{1}{2}\sin 4x + \frac{\sqrt{3}}{2}\cos 4x = \frac{1}{2} \Leftrightarrow \sin(4x + \frac{\pi}{3}) = \sin\frac{\pi}{6}$

$\Leftrightarrow \left[\begin{array}{l}4x + \frac{\pi}{3} = \frac{\pi}{6} + k2\pi\\ 4x + \frac{\pi}{3} = \frac{5\pi}{6} + k2\pi\end{array}\right. \Leftrightarrow \left[\begin{array}{l}4x + \frac{\pi}{3} = \frac{\pi}{6} + k2\pi\\ 4x + \frac{\pi}{3} = \frac{5\pi}{6} + k2\pi\end{array}\right. \Leftrightarrow \left[\begin{array}{l}4x = -\frac{\pi}{6} + k2\pi\\ 4x = \frac{\pi}{2} + k2\pi\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x = -\frac{\pi}{24} + k\frac{\pi}{2}\\ x = \frac{\pi}{8} + k\frac{\pi}{2}\end{array}\right. (k \in Z)$

16) : $\sin 2x + (1 + 2\cos 3x)\sin x - 2\sin^2(2x + \frac{\pi}{4}) = 0$

$\sin 2x + (1+2\cos 3x)\sin x - 2\sin(2x + \frac{\pi}{4}) = 0$

$\Leftrightarrow \sin 2x + \sin x + \sin 4x - \sin 2x = 1 - \cos(4x + \frac{\pi}{2}) \Leftrightarrow \sin x + \sin 4x = 1 + \sin 4x \Leftrightarrow \sin x = 1$

$\Leftrightarrow x = \frac{\pi}{2} + k2\pi, k \in Z$

17) Tìm $x \in (0;\pi)$ thoả mãn ph- ơng trình: $\cot x - 1 = \frac{\cos 2x}{1+\tan x} + \sin^2 x - \frac{1}{2}\sin 2x$.

ĐK: $\begin{cases}\sin 2x \neq 0\\ \sin x+\cos x\neq 0\end{cases} \Leftrightarrow \begin{cases}\sin 2x\neq 0\\ \tan x\neq -1\end{cases}$

Khi đó pt $\Leftrightarrow \dfrac{\cos x-\sin x}{\sin x}=\dfrac{\cos 2x.\cos x}{\cos x+\sin x}+\sin^2 x-\sin x\cos x$

$\Leftrightarrow \dfrac{\cos x-\sin x}{\sin x}=\cos^2 x-\sin x\cos x+\sin^2 x-\sin x\cos x$

$\Leftrightarrow \cos x-\sin x=\sin x(1-\sin 2x)$

$\Leftrightarrow (\cos x-\sin x)(\sin x\cos x-\sin^2 x-1)=0$

$\Leftrightarrow (\cos x-\sin x)(\sin 2x+\cos 2x-3)=0$

$\Leftrightarrow \cos x-\sin x=0 \Leftrightarrow$ tanx = 1 $\Leftrightarrow x=\dfrac{\pi}{4}+k\pi (k\in Z)$ (tm)

$x\in(0;\pi)\Rightarrow k=0\Rightarrow x=\dfrac{\pi}{4}$

17) $1+\sin\dfrac{x}{2}\sin x-\cos\dfrac{x}{2}\sin^2 x=2\cos^2\left(\dfrac{\pi}{4}-\dfrac{x}{2}\right)$

$(1)\Leftrightarrow 1+\sin\dfrac{x}{2}\sin x-\cos\dfrac{x}{2}\sin^2 x=1+\cos\left(\dfrac{\pi}{2}-x\right)=1+\sin x$

$\Leftrightarrow \sin x\left(\sin\dfrac{x}{2}-\cos\dfrac{x}{2}\sin x-1\right)=0\Leftrightarrow \sin x\left(\sin\dfrac{x}{2}-\cos\dfrac{x}{2}.2\sin\dfrac{x}{2}\cos\dfrac{x}{2}-1\right)=0$

$\Leftrightarrow \sin x\left(\sin\dfrac{x}{2}-1\right)\left(2\sin^2\dfrac{x}{2}+2\sin\dfrac{x}{2}+1\right)=0$

$\Leftrightarrow \left[\begin{array}{l}\sin x=0\\ \sin\dfrac{x}{2}=1\\ 2\sin^2\dfrac{x}{2}+2\sin\dfrac{x}{2}+1\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=k\pi\\ \dfrac{x}{2}=\dfrac{\pi}{2}+k2\pi\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=k\pi\\ x=\pi+k4\pi\end{array}\right. \Leftrightarrow x=k\pi, k\in \mathbf{Z}$

18) 2cos4x - ($\sqrt{3}$ - 2)cos2x = sin2x + $\sqrt{3}$ biết x$\in$ [0 ; π].

Phương trình đã cho tương đương với 2(cos4x + cos2x) = $\sqrt{3}$ (cos2x + 1) + sin2x

$\Leftrightarrow 4\cos 3x\cos x=2\sqrt{3}\cos^2 x+2\sin x\cos x \Leftrightarrow \left[\begin{array}{l}\cos x=0\\ 2\cos 3x=\sqrt{3}\cos x+\sin x\end{array}\right.$

+ $\cos x=0 \Leftrightarrow x=\dfrac{\pi}{2}+k\pi$

+ $2\cos 3x=\sqrt{3}\cos x+\sin x \Leftrightarrow \cos 3x=\cos(x-\dfrac{\pi}{6}) \Leftrightarrow \left[\begin{array}{l}3x=x-\dfrac{\pi}{6}+k2\pi\\ 3x=\dfrac{\pi}{6}-x+k2\pi\end{array}\right.$

$\Leftrightarrow \left[\begin{array}{l} x=-\dfrac{\pi}{12}+k\pi \\ x=\dfrac{\pi}{24}+\dfrac{k\pi}{2} \end{array}\right.$ vì $x\in[0;\pi] \Rightarrow x=\dfrac{\pi}{2}, x=\dfrac{11\pi}{12}, x=\dfrac{\pi}{24}, x=\dfrac{13\pi}{24}$

19) $\dfrac{\cos^2 x.(\cos x-1)}{\sin x+\cos x}=2(1+\sin x)$.

ĐK: $\sin x+\cos x\neq 0$

Khi đó $PT \Leftrightarrow (1-\sin^2 x)(\cos x-1)=2(1+\sin x)(\sin x+\cos x)$

$\Leftrightarrow (1+\sin x)(1+\cos x+\sin x+\sin x.\cos x)=0$

$\Leftrightarrow (1+\sin x)(1+\cos x)(1+\sin x)=0$

$\Leftrightarrow \left[\begin{array}{l} \sin x=-1 \\ \cos x=-1 \end{array}\right.$ (thoả mãn điều kiện)

$\Leftrightarrow \left[\begin{array}{l} x=-\dfrac{\pi}{2}+k2\pi \\ x=\pi+m2\pi \end{array}\right.$ $(k,m\in \mathbf{Z})$

Vậy phương trình đã cho có nghiệm là: $x=-\dfrac{\pi}{2}+k2\pi$ **và** $x=\pi+m2\pi$ $(k,m\in \mathbf{Z})$

20) $\sqrt{3}\sin^2 x+\dfrac{1}{2}\sin 2x=\tan x$

* Đk: cosx $\neq$ 0 $\Leftrightarrow$ x $\neq$ $\dfrac{\pi}{2}+k\pi$.

PT đã cho $\Leftrightarrow$ $\sqrt{3}\sin^2 x$ + sinxcosx - $\dfrac{\sin x}{\cos x}$ = 0

* $\Leftrightarrow$ sinx($\sqrt{3}$ sinx + cosx - $\dfrac{1}{\cos x}$) = 0

$\Leftrightarrow \left[\begin{array}{l} \sin x=0 \\ \sqrt{3}\sin x+\cos x-\dfrac{1}{\cos x}=0 \end{array}\right.$

* Sinx = 0 $\Leftrightarrow$ x = kπ.

* $\sqrt{3}$ sinx + cosx - $\dfrac{1}{\cos x}$ = 0 $\Leftrightarrow$ $\sqrt{3}$ tanx + 1 - $\dfrac{1}{\cos^2 x}$ = 0

$\Leftrightarrow$ $\tan^2 x$ - $\sqrt{3}$ tanx = 0 $\Leftrightarrow \left[\begin{array}{l} \tan x=0 \\ \tan x=\sqrt{3} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=k\pi \\ x=\dfrac{\pi}{3}+k\pi \end{array}\right.$

Vậy PT có các họ nghiệm: x = kπ, x = $\dfrac{\pi}{3}+k\pi$

21) $\sqrt{3}\sin 2x.(2\cos x+1)+2=\cos 3x+\cos 2x-3\cos x.$

Pt $\Leftrightarrow \sqrt{3}\sin 2x(2\cos x+1)=(\cos 3x-\cos x)+(\cos 2x-1)-(2\cos x+1)$

$\Leftrightarrow \sqrt{3}\sin 2x(2\cos x+1)=-4\sin^2 x\cos x-2\sin^2 x-(2\cos x+1)$

$\Leftrightarrow (2\cos x+1)(\sqrt{3}\sin 2x+2\sin^2 x+1)=0$

• $\sqrt{3}\sin 2x + 2\sin^2 x + 1 = 0 \Leftrightarrow \sqrt{3}\sin 2x - \cos 2x = -2 \Leftrightarrow \sin(2x - \frac{\pi}{6}) = -1$

$\Leftrightarrow x = -\frac{\pi}{6} + k\pi$

• $2\cos x + 1 = 0 \Leftrightarrow \left[\begin{array}{l} x = \frac{2\pi}{3} + k2\pi \\ x = -\frac{2\pi}{3} + k2\pi \end{array}\right. \quad (k \in Z)$

Vậy phương trình có nghiệm: $x = \frac{2\pi}{3} + k2\pi$; $x = -\frac{2\pi}{3} + k2\pi$ và $x = -\frac{\pi}{6} + k\pi$ $(k \in Z)$

22) $\dfrac{2\sqrt{3}\cos^2 x + 2\sin 3x\cos x - \sin 4x - \sqrt{3}}{\sqrt{3}\sin x + \cos x} = 1$

ĐK: $\sqrt{3}sinx + cosx \neq 0 \Leftrightarrow \cos\left(x - \frac{\pi}{3}\right) \neq 0 \Leftrightarrow x \neq \frac{5\pi}{6} + k\pi, k \in Z$

- Với ĐK trên PT đã cho tương đương với

$\sqrt{3}cos2x + sin2x = \sqrt{3}sinx + cosx \Leftrightarrow \cos\left(2x - \frac{\pi}{6}\right) = \cos\left(x - \frac{\pi}{3}\right)$

$\Leftrightarrow \left[\begin{array}{l} \left(2x - \frac{\pi}{6}\right) = \left(x - \frac{\pi}{3}\right) + k2\pi \\ \left(2x - \frac{\pi}{6}\right) = -\left(x - \frac{\pi}{3}\right) + k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = -\frac{\pi}{6} + k2\pi \\ x = \frac{\pi}{6} + \frac{k2\pi}{3} \end{array}\right. ; k \in Z$

Đối chiếu ĐK ta được nghiệm của pt đã cho là $x = \frac{\pi}{6} + k2\pi, x = \frac{3\pi}{2} + k2\pi, k \in Z$

23) $\dfrac{1 - 2\cos^2 x}{\sin x.\cos x} + 2\tan 2x + \cot^3 4x = 3$

+) ĐK: $\sin 4x \neq 0$

+) PT $\Leftrightarrow \cot^3 4x - 4\cot 4x - 3 = 0$

$\Leftrightarrow \left[\begin{array}{l} \cot 4x = 1 \\ \cot 4x = \frac{1 \pm \sqrt{13}}{2} \end{array}\right.$

24) $\tan x = \sqrt{2}\cos x\cos\left(x - \frac{\pi}{4}\right)$

ĐK: $x \neq l\pi$ ($l \in$ □)

PT $\Leftrightarrow$ tanx = cosx(sinx + cosx) $\Leftrightarrow$ sinx = $\cos^2$x(sinx + cosx)

$\Leftrightarrow$ sinx($\sin^2$x + $\cos^2$x) = $\cos^2$x(sinx + cosx)

$\Leftrightarrow$ $\sin^3$x = $\cos^3$x $\Leftrightarrow$ sinx = cosx $\Leftrightarrow$ $x = \frac{\pi}{4} + k\pi$ (k ∈ □) (Thoả mãn)

25) $\dfrac{\left(2 - \sqrt{3}\right)\cos 2x - 2\sin^2\left(x - \frac{13\pi}{4}\right)}{4\sin^2 x - 1} = -1$

Đk $4\sin^2 x - 1 \neq 0 \Leftrightarrow \cos 2x \neq \frac{1}{2} \Leftrightarrow x \neq \pm\frac{\pi}{6} + k\pi, k \in$ □

Phương trình đã cho tương đương với $\left(2-\sqrt{3}\right)\cos 2x-1+\cos\left(2x-\frac{\pi}{2}\right)=2\cos 2x-1$

$\Leftrightarrow \sin 2x-\sqrt{3}\cos 2x=0 \Leftrightarrow \tan 2x=\sqrt{3} \Leftrightarrow 2x=\frac{\pi}{3}+k\pi \Leftrightarrow x=\frac{\pi}{6}+k\frac{\pi}{2}, k\in\mathbb{Z}$.

Kết hợp với điều kiện ta có $\left[\begin{array}{l} x=\frac{2\pi}{3}+k2\pi \\ x=\frac{5\pi}{3}+k2\pi \end{array}\right., k\in\mathbb{Z}$.

26) $\frac{5\sin 2x-4\left(\sin^4 x+\cos^4 x\right)+6}{2\cos 2x+\sqrt{3}}=0$

Điều kiện: $2\cos 2x+\sqrt{3}\neq 0 \Leftrightarrow 2x\neq\pm\frac{5\pi}{6}+k2\pi \Leftrightarrow x\neq\pm\frac{5\pi}{12}+k\pi, k\in Z$

$(1)\Leftrightarrow 5\sin 2x-4\left(1-\frac{1}{2}\sin^2 2x\right)+6=0$

$\Leftrightarrow 2\sin^2 x+5\sin 2x+2=0(2)$

Đặt sin2x=t, Đk: $|t|\leq 1$ $(2)\Leftrightarrow 2t^2+5t+2=0 \Leftrightarrow \left[\begin{array}{l} t=-2(loai) \\ t=-\frac{1}{2}(TM) \end{array}\right.$

Khi t=1/2=>sin2x=-1/2 $\Leftrightarrow \left[\begin{array}{l} 2x=-\frac{\pi}{6}+k2\pi \\ 2x=\frac{7\pi}{6}+k2\pi \end{array}\right., k\in Z \Leftrightarrow \left[\begin{array}{l} x=-\frac{\pi}{12}+k2\pi(tm) \\ x=\frac{7\pi}{12}+k2\pi(l) \end{array}\right., k\in Z$

27) $2\sin^2 x+2\sqrt{3}\sin x\cos x+1=3\left(\cos x+\sqrt{3}\sin x\right)$

$2+\sqrt{3}\sin 2x-\cos 2x=3\left(\cos x+\sqrt{3}\sin x\right)\Leftrightarrow 1+\left(\frac{\sqrt{3}}{2}\sin 2x-\frac{1}{2}\cos 2x\right)=3\left(\frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x\right)$

$\Leftrightarrow 1+\cos\left(2x-\frac{2\pi}{3}\right)=3\cos\left(x-\frac{\pi}{3}\right)\Leftrightarrow 2\cos^2\left(x-\frac{\pi}{3}\right)=3\cos\left(x-\frac{\pi}{3}\right)$

$\Leftrightarrow \cos\left(x-\frac{\pi}{3}\right)=0 \Leftrightarrow x-\frac{\pi}{3}=\frac{\pi}{2}+k\pi \Leftrightarrow x=\frac{5\pi}{6}+k\pi$

28) $4\cos^4 x-4\sqrt{3}\cos^3 x+\cos^2 x+\sqrt{3}\sin 2x+3=0$

$$\Leftrightarrow \left(4\cos^4 x - 4\sqrt{3}\cos^3 x + 3\cos^2 x\right) + \left(\cos^2 x + 2\sqrt{3}\sin x.\cos x + 3\sin^2 x\right) = 0$$

$$\Leftrightarrow \left(2\cos^2 x - \sqrt{3}\cos x\right)^2 + \left(\cos x + \sqrt{3}\sin x\right)^2 = 0$$

$$\Leftrightarrow \cos^2 x\left(2\cos x - \sqrt{3}\right)^2 + 4\cos^2\left(x - \frac{\pi}{3}\right) = 0$$

$$\Leftrightarrow \left[\begin{array}{l} \begin{cases} \cos^2 x = 0 \\ \cos^2\left(x - \frac{\pi}{3}\right) = 0 \end{cases} \\ \begin{cases} \cos x = \frac{\sqrt{3}}{2} \\ \cos^2\left(x - \frac{\pi}{3}\right) = 0 \end{cases} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} vo \quad no \\ \begin{cases} x = \pm\frac{\pi}{6} + k2\pi \\ x = -\frac{\pi}{6} + l\pi \end{cases} \end{array}\right.$$

$$\Leftrightarrow x = -\frac{\pi}{6} + k2\pi, k \in \mathbb{Z}$$

29) $1 + \sin\frac{x}{2}\sin x - \cos\frac{x}{2}\sin^2 x = 2\cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right)$

$$1 + \sin\frac{x}{2}\sin x - \cos\frac{x}{2}\sin^2 x = 2\cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right) \qquad (1)$$

$$(1) \Leftrightarrow 1 + \sin\frac{x}{2}\sin x - \cos\frac{x}{2}\sin^2 x = 1 + \cos\left(\frac{\pi}{2} - x\right) = 1 + \sin x$$

$$\Leftrightarrow \sin x\left(\sin\frac{x}{2} - \cos\frac{x}{2}\sin x - 1\right) = 0 \Leftrightarrow \sin x\left(\sin\frac{x}{2} - \cos\frac{x}{2}.2\sin\frac{x}{2}\cos\frac{x}{2} - 1\right) = 0$$

$$\Leftrightarrow \sin x\left(\sin\frac{x}{2} - 1\right)\left(2\sin^2\frac{x}{2} + 2\sin\frac{x}{2} + 1\right) = 0$$

$$\Leftrightarrow \left[\begin{array}{l} \sin x = 0 \\ \sin\frac{x}{2} = 1 \\ 2\sin^2\frac{x}{2} + 2\sin\frac{x}{2} + 1 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = k\pi \\ \frac{x}{2} = \frac{\pi}{2} + k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = k\pi \\ x = \pi + k4\pi \end{array}\right. \Leftrightarrow x = k\pi, k \in \mathbf{Z}$$

30) $2\cos x + \frac{1}{3}\cos^2(x + 3\pi) = \frac{8}{3} + \sin 2(x - \pi) + 3\cos(x + 10,5\pi) + \frac{1}{3}\sin^2 x$

TXĐ: $\mathbb{R}$; Trên đó PT đó cho tương đương với PT $6\cos x + \cos^2 x = 8 + 3\sin 2x - 9\sin x + \sin^2 x$ (1)

$(1) \Leftrightarrow 6\cos x - 6\sin x\cos x + \cos^2 x - \sin^2 x + 9\sin x - 8 = 0 \Leftrightarrow 6\cos x(1 - \sin x) + 2 - 2\sin^2 x + 9\sin x - 9 = 0$

$\Leftrightarrow (1 - \sin x)(6\cos x + 2\sin x - 7) = 0$

$\sin x = 1 \Leftrightarrow x = \frac{\pi}{2} + k2\pi \quad (k \in \mathbb{Z})$

PT 6cosx + 2sinx - 7 = 0 v« nghiÖm v× $6^2 + 2^2 < 7^2$. VËy nghiÖm cña PT ®· cho lµ

$$x = \frac{\pi}{2} + k2\pi \quad (k \in \mathbb{Z})$$

31) $8\left(\sin^6 x + cos^6 x\right) + 3\sqrt{3}\sin 4x = 3\sqrt{3}cos 2x - 9\sin 2x + 11$

$$\left(\sin^6 x + cos^6 x\right) = 1 - \frac{3}{4}\sin^2 2x \quad (1)$$

Thay (1) vµo ph-¬ng tr×nh (*) ta cã :

$$8\left(\sin^6 x + cos^6 x\right) + 3\sqrt{3}\sin 4x = 3\sqrt{3}cos2x - 9\sin 2x + 11$$

$$\Leftrightarrow 8\left(1 - \frac{3}{4}\sin^2 2x\right) + 3\sqrt{3}\sin 4x = 3\sqrt{3}cos2x - 9\sin 2x + 11$$

$$\Leftrightarrow 3\sqrt{3}\sin 4x - 3\sqrt{3}cos2x = 6\sin^2 2x - 9\sin 2x + 3$$

$$\Leftrightarrow \sqrt{3}\sin 4x - \sqrt{3}cos2x = 2\sin^2 2x - 3\sin 2x + 1$$

$$\Leftrightarrow \sqrt{3}cos2x.\left(2\sin 2x - 1\right) = (2\sin 2x - 1)(\sin 2x - 1)$$

$$\Leftrightarrow \left(2\sin 2x - 1\right)\left(\sqrt{3}cos2x - \sin 2x + 1\right) = 0$$

$$\Leftrightarrow \left[\begin{array}{l} 2\sin 2x - 1 = 0 \\ \sqrt{3}cos2x - \sin 2x + 1 = 0 \end{array}\right. \Leftrightarrow \left[\begin{array}{ll} 2\sin 2x = 1 & (2) \\ \sin 2x - \sqrt{3}cos2x = 1 & (3) \end{array}\right.$$

Giải (2) : $\left[\begin{array}{l} x = \frac{\Pi}{12} + k\Pi \\ x = \frac{5\Pi}{12} + k\Pi \end{array}\right. (k \in Z)$; Giải (3) $\left[\begin{array}{l} x = \frac{\Pi}{4} + k\Pi \\ x = \frac{7\Pi}{12} + k\Pi \end{array}\right. (k \in Z)$

Kết luận :

32) $\sin^2 x + \frac{1 + \text{sinx}}{\text{cosx}} - \frac{1}{2}\sin 2x = cosx$

ĐK: cosx $\neq 0$. PT $\Leftrightarrow$ (1 + sinx + cosx)sin^{2}x = 0 nghiệm x = k π

33) : $\tan 3x - 2\tan 4x + \tan 5x = 0$ với $x \in (0; 2\pi)$.

ĐK: $\cos 3x \neq 0; \cos 4x \neq 0; \cos 5x \neq 0$.

Phương trình cho

$$\Leftrightarrow \frac{\sin 8x}{\cos 3x.\cos 5x} - \frac{2\sin 4x}{\cos 4x} = 0 \qquad \Leftrightarrow 2\sin 4x\left(\frac{\cos^2 4x - \cos 3x.\cos 5x}{\cos 3x.\cos 4x.\cos 5x}\right) = 0$$

$$\Leftrightarrow \sin 4x\left(\frac{1 + \cos 8x - \cos 2x - \cos 8x}{\cos 3x.\cos 4x.\cos 5x}\right) = 0 \qquad \Leftrightarrow \sin 4x\left(\frac{2\sin^2 x}{\cos 3x.\cos 4x.\cos 5x}\right) = 0$$

$$\Leftrightarrow \left[\begin{array}{l} \sin 4x = 0 \\ \sin x = 0 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = k\frac{\pi}{4}, k \in \mathbb{Z} \\ x = k\pi \end{array}\right. \Leftrightarrow x = k\frac{\pi}{4}, k \in \mathbb{Z}$$

Do $x \in (0; 2\pi)$ nên phương trình cho có nghiệm là

$$x = \frac{\pi}{4}; x = \pi; x = \frac{5\pi}{4}; x = \frac{3\pi}{2}; x = \frac{7\pi}{4}$$

34) $2\sin^2 x + 2\sqrt{3}\sin x\cos x + 1 = 3\left(\cos x + \sqrt{3}\sin x\right)$

$$2 + \sqrt{3}\sin 2x - \cos 2x = 3\left(\cos x + \sqrt{3}\sin x\right) \Leftrightarrow 1 + \left(\frac{\sqrt{3}}{2}\sin 2x - \frac{1}{2}\cos 2x\right) = 3\left(\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right)$$

$$\Leftrightarrow 1 + \cos\left(2x - \frac{2\pi}{3}\right) = 3\cos\left(x - \frac{\pi}{3}\right) \Leftrightarrow 2\cos^2\left(x - \frac{\pi}{3}\right) = 3\cos\left(x - \frac{\pi}{3}\right)$$

$\Leftrightarrow \cos\left(x-\frac{\pi}{3}\right)=0 \Leftrightarrow x-\frac{\pi}{3}=\frac{\pi}{2}+k\pi \Leftrightarrow x=\frac{5\pi}{6}+k\pi$

35) $\sin 2x-\cos 2x+3\sin x+5\cos x-4=0$

+Ph- ong trình $\Leftrightarrow 2\sin^2 x+3\sin x-5+\cos x(2\sin x+5)=0$

$\Leftrightarrow (\sin x-1)(2\sin x+5)+\cos x(2\sin x+5)=0$

$\Leftrightarrow (2\sin x+5)(\sin x+\cos x-1)=0$

$$\Leftrightarrow \left[\begin{array}{l}\sin x=-\frac{5}{2}(l)\\ \sin x+\cos x=1\end{array}\right. \Leftrightarrow \sin(x+\frac{\pi}{4})=\frac{1}{\sqrt{2}} \Leftrightarrow \left[\begin{array}{l}x+\frac{\pi}{4}=\frac{\pi}{4}+k2\pi\\ x+\frac{\pi}{4}=\frac{3\pi}{4}+k2\pi\end{array}\right. \quad (k\in Z)$$

$$\Leftrightarrow \left[\begin{array}{l}x=k2\pi\\ x=\frac{\pi}{2}+k2\pi\end{array}\right.$$

+Vậy ph- ơng trình có nghiệm $x=k\pi$; $x=\frac{\pi}{2}+k2\pi$

36) $\frac{1}{\tan x+\cot 2x}=\frac{\sqrt{2}(\cos x-\sin x)}{\cot x-1}$

Điều kiện:sinx.cosx $\neq 0$ và cotx $\neq 1$

Phơng trình tơng đơng

$$\frac{1}{\frac{\sin x}{\cos x}+\frac{\cos 2x}{\sin 2x}}=\frac{\sqrt{2}(\cos x-\sin x)}{\frac{\cos x}{\sin x}-1}$$

$\Rightarrow \cos x=\frac{\sqrt{2}}{2} \Rightarrow x=\pm\frac{\pi}{4}+k2\pi$

Đối chiếu điều kiện pt có 1 họ nghiệm x $=-\frac{\pi}{4}+k2\pi$

37) $2\cos^2\left(\frac{\pi}{4}-2x\right)+\sqrt{3}\cos 4x=4\cos^2 x-1$

Phương trình tương đương với $\Leftrightarrow 1+\cos\left(\frac{\pi}{2}-4x\right)+\sqrt{3}\cos 4x=4\cos^2 x-1$

$\Leftrightarrow \sin 4x+\sqrt{3}\cos 4x=2\left(2\cos^2 x-1\right) \Leftrightarrow \frac{1}{2}\sin 4x+\frac{\sqrt{3}}{2}\cos 4x=\cos 2x \Leftrightarrow \cos\left(4x-\frac{\pi}{6}\right)=\cos 2x$

$$\Leftrightarrow \boxed{\left[\begin{array}{l}x=\frac{\pi}{12}+k\pi\\ x=\frac{\pi}{36}+\frac{k\pi}{3}\end{array}\right.(k\in \mathbb{Z})}$$

38) $2\cos x+\frac{1}{3}\cos^2(\pi+x)=\frac{8}{3}+\sin 2x+3\cos(x+\frac{\pi}{2})+\frac{1}{3}\sin^2 x$

$\Leftrightarrow 2\cos x+\frac{1}{3}\cos^2 x=\frac{8}{3}+\sin 2x-3\sin x+\frac{1}{3}\sin^2 x \Leftrightarrow 6\cos x+\cos^2 x=8+6\sin x.\cos x-9\sin x+\sin^2 x$

$\Leftrightarrow 6\cos x(1-\sin x)-(2\sin^2 x-9\sin x+7)=0 \Leftrightarrow 6\cos x(1-\sin x)-2(\sin x-1)(\sin x-\frac{7}{2})=0$

$\Leftrightarrow (1-\sin x)(6\cos x-2\sin x+7)=0 \Leftrightarrow \begin{cases}1-\sin x=0 _{(1)}\\ 6\cos x-2\sin x+7=0_{(2)}\end{cases} \Leftrightarrow x=\frac{\pi}{2}+k2\pi;(k\in Z)$

(p/t (2) vô nghiệm)

39) $\sin 2x-2\sqrt{2}(\sin x+\cos x)=5$

Đặt sinx + cosx = t ($|t|\le\sqrt{2}$). $\Rightarrow$ sin2x = t^2 - 1 $\Rightarrow$ (I)

$\Leftrightarrow t^2-2\sqrt{2}t-6=0 \Leftrightarrow t=-\sqrt{2}$)

+Giải được phương trình sinx + cosx = $-\sqrt{2}$ … $\Leftrightarrow$ $\cos(x-\frac{\pi}{4})=-1$

+ Lấy nghiệm Kết luận : $x=\frac{5\pi}{4}+k2\pi$ ($k\in \mathbf{Z}$) hoặc dưới dạng đúng khác

40) Tìm $x\in(0;\pi)$ thoả mãn ph- ơng trình: cot x - 1 = $\frac{\cos 2x}{1+\tan x}+\sin^2 x-\frac{1}{2}\sin 2x$.

ĐK: $\begin{cases}\sin 2x\neq 0\\ \sin x+\cos x\neq 0\end{cases} \Leftrightarrow \begin{cases}\sin 2x\neq 0\\ \tan x\neq -1\end{cases}$

Khi đó pt $\Leftrightarrow \frac{\cos x-\sin x}{\sin x}=\frac{\cos 2x.\cos x}{\cos x+\sin x}+\sin^2 x-\sin x\cos x$

$\Leftrightarrow \frac{\cos x-\sin x}{\sin x}=\cos^2 x-\sin x\cos x+\sin^2 x-\sin x\cos x$

$\Leftrightarrow \cos x-\sin x=\sin x(1-\sin 2x)$

$\Leftrightarrow (\cos x-\sin x)(\sin x\cos x-\sin^2 x-1)=0$

$\Leftrightarrow (\cos x-\sin x)(\sin 2x+\cos 2x-3)=0$

$\Leftrightarrow \cos x-\sin x=0 \Leftrightarrow$ tanx = 1 $\Leftrightarrow x=\frac{\pi}{4}+k\pi(k\in Z)$ (tm)

$x\in(0;\pi)\Rightarrow k=0\Rightarrow x=\frac{\pi}{4}$

41) $\sqrt{3}$ (2cos^{2}x + cosx – 2) + (3 – 2cosx)sinx = 0

Phương trình đã cho tương đương với phương trình :

$\left(2\sin x-\sqrt{3}\right)\left(\sqrt{3}\sin x+\cos x\right)=0 \Leftrightarrow \left[\begin{array}{l}\sin x=\frac{\sqrt{3}}{2}\\ \sqrt{3}\sin x+\cos x=0\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=(-1)^n\frac{\pi}{3}+n\pi,\ n\in \mathbb{Z}\\ x=-\frac{\pi}{6}+k\pi,\ k\in \mathbb{Z}\end{array}\right.$

42) $\cos x+\cos 3x=1+\sqrt{2}\sin\left(2x+\frac{\pi}{4}\right)$

1. $\cos x+\cos 3x=1+\sqrt{2}\sin\left(2x+\frac{\pi}{4}\right)\Leftrightarrow 2\cos x\cos 2x=1+\sin 2x+\cos 2x$

$\Leftrightarrow 2\cos^2 x+2\sin x\cos x-2\cos x\cos 2x=0\Leftrightarrow \cos x\left(\cos x+\sin x-\cos 2x\right)=0$

$$\Leftrightarrow \cos x\left(\cos x+\sin x\right)\left(1+\sin x-\cos x\right)=0\Leftrightarrow\begin{cases}\cos x=0\\ \cos x+\sin x=0\\ 1+\sin x-\cos x=0\end{cases}\Leftrightarrow\begin{cases}x=\frac{\pi}{2}+k\pi\\ x=-\frac{\pi}{4}+k\pi\\ \sin\left(x-\frac{\pi}{4}\right)=-\frac{1}{\sqrt{2}}\end{cases}$$

$$\Leftrightarrow\begin{cases}x=\frac{\pi}{2}+k\pi\\ x=-\frac{\pi}{4}+k\pi\\ x-\frac{\pi}{4}=-\frac{\pi}{4}+k2\pi\\ x-\frac{\pi}{4}=\frac{5\pi}{4}+k2\pi\end{cases}\Leftrightarrow\begin{cases}x=\frac{\pi}{2}+k\pi\\ x=-\frac{\pi}{4}+k\pi\\ x=k2\pi\end{cases}$$

43) $$\frac{\sin^4 2x+\cos^4 2x}{\tan(\frac{\pi}{4}-x).\tan(\frac{\pi}{4}+x)}=\cos^4 4x$$

+) ĐK: $x\neq\frac{\pi}{4}+k\frac{\pi}{2},k\in Z$

+) $\tan(\frac{\pi}{4}-x)\tan(\frac{\pi}{4}+x)=\tan(\frac{\pi}{4}-x)\cot(\frac{\pi}{4}-x)=1$

$\sin^4 2x+\cos^4 2x=1-\frac{1}{2}\sin^2 4x=\frac{1}{2}+\frac{1}{2}\cos^2 4x$

$pt\Leftrightarrow 2\cos^4 4x-\cos^2 4x-1=0$

+) Giải pt được $\cos^2 4x = 1$ $\Leftrightarrow \cos 8x = 1$ $\Leftrightarrow$ $x=k\frac{\pi}{4}$ và $\cos^2 4x = -1/2$ (VN)

+) Kết hợp ĐK ta được nghiệm của phương trình là $x=k\frac{\pi}{2},k\in Z$

44) $1+\sin\frac{x}{2}\sin x-\cos\frac{x}{2}\sin^2 x=2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)$

$$1+\sin\frac{x}{2}\sin x-\cos\frac{x}{2}\sin^2 x=2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)\qquad(1)$$

$(1)\Leftrightarrow 1+\sin\frac{x}{2}\sin x-\cos\frac{x}{2}\sin^2 x=1+\cos\left(\frac{\pi}{2}-x\right)=1+\sin x$

$\Leftrightarrow \sin x\left(\sin\frac{x}{2}-\cos\frac{x}{2}\sin x-1\right)=0\Leftrightarrow \sin x\left(\sin\frac{x}{2}-\cos\frac{x}{2}.2\sin\frac{x}{2}\cos\frac{x}{2}-1\right)=0$

$\Leftrightarrow \sin x\left(\sin\frac{x}{2}-1\right)\left(2\sin^2\frac{x}{2}+2\sin\frac{x}{2}+1\right)=0$

$$\Leftrightarrow \left[\begin{array}{l} \sin x = 0 \\ \sin\frac{x}{2} = 1 \\ 2\sin^2\frac{x}{2} + 2\sin\frac{x}{2} + 1 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = k\pi \\ \frac{x}{2} = \frac{\pi}{2} + k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = k\pi \\ x = \pi + k4\pi \end{array}\right. \Leftrightarrow x = k\pi, k \in \mathbf{Z}$$

45) $\frac{1}{\tan x + \cot 2x} = \frac{\sqrt{2}\left(\cos x - \sin x\right)}{\cot x - 1}$

$$\begin{cases} \cos x.\sin 2x.\sin x.\left(\tan x + \cot 2x\right) \neq 0 \\ \cot x \neq 1 \end{cases}$$

Từ (1) ta có: $\frac{1}{\frac{\sin x}{\cos x} + \frac{\cos 2x}{\sin 2x}} = \frac{\sqrt{2}\left(\cos x - \sin x\right)}{\frac{\cos x}{\sin x} - 1} \Leftrightarrow \frac{\cos x.\sin 2x}{\cos x} = \sqrt{2}\sin x$

$\Leftrightarrow 2\sin x.\cos x = \sqrt{2}\sin x$

$$\Leftrightarrow \cos x = \frac{\sqrt{2}}{2} \Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{4} + k2\pi \\ x = -\frac{\pi}{4} + k2\pi \end{array}\right. \left(k \in \mathbb{Z}\right)$$

Giao với điều kiện, ta được họ nghiệm của phương trình đã cho là $x = -\frac{\pi}{4} + k2\pi \left(k \in \mathbb{Z}\right)$

46) $1 + \tan 2x = \frac{\left(\sin x - \cos x\right)^2}{\cos^2 2x}$

47) $\sin x + \sin^2 x + \sin^3 x + \sin^4 x = \cos x + \cos^2 x + \cos^3 x + \cos^4 x$

TXĐ: D =R

$\sin x + \sin^2 x + \sin^3 x + \sin^4 x = \cos x + \cos^2 x + \cos^3 x + \cos^4 x$

$$\Leftrightarrow (\sin x - cosx).\left[2 + 2(\sin x + cosx) + \sin x.cosx\right] = 0 \Leftrightarrow \left[\begin{array}{l} \sin x - cosx = 0 \\ 2 + 2(\sin x + cosx) + \sin x.cosx = 0 \end{array}\right.$$

+ Với $\sin x - cosx = 0 \Leftrightarrow x = \frac{\pi}{4} + k\pi \quad (k \in Z)$

+ Với $2 + 2(\sin x + cosx) + \sin x.cosx = 0$, đặt t = $\sin x + cosx \quad (t \in \left[-\sqrt{2};\sqrt{2}\right])$

được pt : $t^2 + 4t + 3 = 0 \Leftrightarrow \left[\begin{array}{l} t = -1 \\ t = -3(loai) \end{array}\right.$

$$t = -1 \Rightarrow \left[\begin{array}{l} x = \pi + m2\pi \\ x = -\frac{\pi}{2} + m2\pi \end{array}\right. (m \in Z) \quad \left[\begin{array}{ll} x = \frac{\pi}{4} + k\pi & (k \in Z) \\ x = \pi + m2\pi & (m \in Z) \\ x = -\frac{\pi}{2} + m2\pi & \end{array}\right.$$

48) (2cosx-1)(2sinx+cosx)=sin2x-sinx

$\Leftrightarrow$ (2cosx-1)(sinx+cosx)=0

$\Leftrightarrow \left[\begin{array}{ll} 2\cos x - 1 = 0 & (1) \\ \sin x + \cos x = 0 & (2) \end{array}\right.$

$(1) \Leftrightarrow \cos x = \frac{1}{2} \Leftrightarrow x = \pm\frac{\pi}{3} + k.2\pi$

$(2) \Leftrightarrow \tan x = -1 \Leftrightarrow x = -\frac{\pi}{4} + k\pi \quad (k \in Z)$

Vậy nghiệm cña phương trình lµ $x = \pm\frac{\pi}{3} + k.2\pi$, $x = -\frac{\pi}{4} + k\pi \quad (k \in Z)$

49) $\cos x + \cos 3x = 1 + \sqrt{2}\sin(2x + \frac{\pi}{4})$

50) $\cot x - 1 = \frac{\cos 2x}{1 + \tan x} + \sin^2 x - \frac{1}{2}\sin 2x$.

□K: $\begin{cases} \sin 2x \neq 0 \\ \sin x + \cos x \neq 0 \end{cases} \Leftrightarrow \begin{cases} \sin 2x \neq 0 \\ \tan x \neq -1 \end{cases}$

PT $\Leftrightarrow \frac{\cos x - \sin x}{\sin x} = \frac{\cos 2x.\cos x}{\cos x + \sin x} + \sin^2 x - \sin x\cos x$

$\Leftrightarrow \frac{\cos x - \sin x}{\sin x} = \cos^2 x - \sin x\cos x + \sin^2 x - \sin x\cos x$

$\Leftrightarrow \cos x - \sin x = \sin x(1 - \sin 2x)$ $\Leftrightarrow (\cos x - \sin x)(\sin x\cos x - \sin^2 x - 1) = 0$

$\Leftrightarrow (\cos x - \sin x)(\sin 2x + \cos 2x - 3) = 0$

$\Leftrightarrow (\cos x - \sin x)(\sqrt{2}\sin(2x + \frac{\pi}{4}) - 3) = 0 \Leftrightarrow \left[\begin{array}{l} \cos x - \sin x = 0 \\ \sqrt{2}\sin(2x + \frac{\pi}{4}) = 3 (voly) \end{array}\right.$

$x \in (0;\pi) \Rightarrow k = 0 \Rightarrow x = \frac{\pi}{4}$

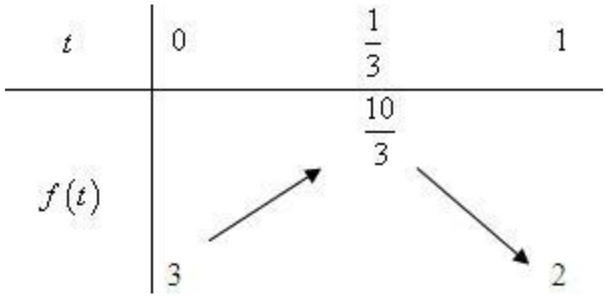
51) Tìm m để phương trình $2\left(\sin^4 x + \cos^4 x\right) + \cos 4x + 2\sin 2x - m = 0$ có nghiệm trên $\left[0;\frac{\pi}{2}\right]$.

Do đó $(1) \Leftrightarrow -3\sin^2 2x + 2\sin 2x + 3 = m$.

Đặt $t = \sin 2x$. Ta có $x \in \left[0;\frac{\pi}{2}\right] \Rightarrow 2x \in [0;\pi] \Rightarrow t \in [0;1]$.

Suy ra $f(t) = -3t^2 + 2t + 3 = m, t \in [0;1]$

Ta có bảng biến thiên



Từ đó phương trình đã cho có nghiệm trên $\left[0;\frac{\pi}{2}\right] \Leftrightarrow 2 \le m \le \frac{10}{3}$

52) $\sqrt{3}\sin^2 x + \frac{1}{2}\sin 2x = \tan x$

* Đk: $\cos x \neq 0 \Leftrightarrow x \neq \frac{\pi}{2} + k\pi$.

PT đã cho $\Leftrightarrow \sqrt{3}\sin^2 x + \sin x\cos x - \frac{\sin x}{\cos x} = 0$

* $\Leftrightarrow \sin x\left(\sqrt{3}\sin x + \cos x - \frac{1}{\cos x}\right) = 0$

$$\Leftrightarrow \left[\begin{array}{l} \sin x = 0 \\ \sqrt{3}\sin x + \cos x - \frac{1}{\cos x} = 0 \end{array}\right.$$

* $\sin x = 0 \Leftrightarrow x = k\pi$.

* $\sqrt{3}\sin x + \cos x - \frac{1}{\cos x} = 0 \Leftrightarrow \sqrt{3}\tan x + 1 - \frac{1}{\cos^2 x} = 0$

$$\Leftrightarrow \tan^2 x - \sqrt{3}\tan x = 0 \Leftrightarrow \left[\begin{array}{l} \tan x = 0 \\ \tan x = \sqrt{3} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = k\pi \\ x = \frac{\pi}{3} + k\pi \end{array}\right.$$

Vậy PT có các họ nghiệm: $x = k\pi$, $x = \frac{\pi}{3} + k\pi$

53) $\sqrt{3}\sin 2x.(2\cos x + 1) + 2 = \cos 3x + \cos 2x - 3\cos x.$

Pt $\Leftrightarrow \sqrt{3}\sin 2x(2\cos x + 1) = (\cos 3x - \cos x) + (\cos 2x - 1) - (2\cos x + 1)$

$\Leftrightarrow \sqrt{3}\sin 2x(2\cos x + 1) = -4\sin^2 x\cos x - 2\sin^2 x - (2\cos x + 1)$

$\Leftrightarrow (2\cos x + 1)(\sqrt{3}\sin 2x + 2\sin^2 x + 1) = 0$

$\sqrt{3}\sin 2x + 2\sin^2 x + 1 = 0 \Leftrightarrow \sqrt{3}\sin 2x - \cos 2x = -2 \Leftrightarrow \sin\left(2x - \frac{\pi}{6}\right) = -1$

$\Leftrightarrow x = -\frac{\pi}{6} + k\pi$

$$\bullet\ 2\cos x + 1 = 0 \Leftrightarrow \left[\begin{array}{l} x = \frac{2\pi}{3} + k2\pi \\ x = -\frac{2\pi}{3} + k2\pi \end{array}\right. \quad (k \in Z)$$

Vậy phương trình có nghiệm: $x = \frac{2\pi}{3} + k2\pi$; $x = -\frac{2\pi}{3} + k2\pi$ và $x = -\frac{\pi}{6} + k\pi$ $(k \in Z)$

54) $2\cos 5x.\cos 3x + \sin x = \cos 8x$, (x ∈ R)

PT ⇔ cos2x + cos8x + sinx = cos8x

⇔ 1- 2sin²x + sinx = 0 ⇔ sinx = 1 v $\sin x = -\frac{1}{2}$

$\Leftrightarrow x = \frac{\pi}{2} + k2\pi; x = -\frac{\pi}{6} + k2\pi; x = \frac{7\pi}{6} + k2\pi, (k \in Z)$

55) $\cos 2x + 2\sin x - 1 - 2\sin x \cos 2x = 0\,(1)$

$$(1) \Leftrightarrow \cos 2x(1 - 2\sin x) - (1 - 2\sin x) = 0$$

$$\Leftrightarrow (\cos 2x - 1)(1 - 2\sin x) = 0$$

Khi cos2x=1<=> $x = k\pi$, k ∈ Z

Khi $\sin x = \frac{1}{2} \Leftrightarrow x = \frac{\pi}{6} + k2\pi$ hoặc $x = \frac{5\pi}{6} + k2\pi$, k ∈ Z

55) **Tìm các nghiệm trên** $(0; 2\pi)$ **của phương trình :** $\frac{\sin 3x - \sin x}{\sqrt{1 - \cos 2x}} = \sin 2x + \cos 2x$

$\frac{\sin 3x - \sin x}{\sqrt{1 - \cos 2x}} = \sin 2x + \cos 2x$ **(1)** $\Leftrightarrow \frac{2\cos 2x.\sin x}{\sqrt{2}|\sin x|} = \sqrt{2}\cos\left(2x - \frac{\pi}{4}\right)$

ĐK : sinx ≠ 0 $\Leftrightarrow x \neq k\pi$

- **Khi** $x \in (0; \pi)$ **thì sinx > 0 nên :**

(1) $\Leftrightarrow \sqrt{2}$ **cos2x =** $\sqrt{2}$ **cos** $\left(2x - \frac{\pi}{4}\right)$

$\Leftrightarrow x = \frac{\pi}{16} + \frac{k\pi}{2}$

Do $x \in (0; \pi)$ **nên** $x = \frac{\pi}{16}$ hay $x = \frac{9\pi}{16}$

- **Khi** $x \in (\pi; 2\pi)$ **thì sinx < 0 nên :**

(1) $\Leftrightarrow -\sqrt{2}\pi$ **cos2x =** $\sqrt{2}$ **cos** $\left(2x - \frac{\pi}{4}\right) \Leftrightarrow \cos(\pi\text{-}2x) = \cos\left(2x\text{-}\frac{\pi}{4}\right)$

$\Leftrightarrow x = \frac{5\pi}{16} + \frac{k\pi}{2}$

Do $x \in (\pi; 2\pi)$ **nên** $x = \frac{21\pi}{16}$ hay $x = \frac{29\pi}{16}$

56) $5\cos 3\left(x + \frac{\pi}{6}\right) + 3\cos 5\left(x - \frac{\pi}{10}\right) = 0$

Pt $\leftrightarrow 5\cos\left(3x + \frac{\pi}{2}\right) + 3\cos\left(5x - \frac{\pi}{2}\right) = 0 \leftrightarrow 5\sin 3x = 3\sin 5x \leftrightarrow 2\sin 3x = 3(\sin 5x - \sin 3x)$

$$\leftrightarrow 2\sin x(3\cos 4x + 4\sin^2 x - 3) = 0 \leftrightarrow \left[\begin{array}{l} \sin x = 0 \\ 3\cos^2 2x - \cos 2x - 2 = 0 \end{array}\right.$$

$$\leftrightarrow \left[\begin{array}{l} x = k\pi \\ x = \pm\frac{1}{2}\arccos(-\frac{2}{3}) + k\pi \end{array}\right. \quad (k \in Z)$$

57) $\sin(2x+\frac{17\pi}{2})+16=2\sqrt{3}.\sin x\cos x+20\sin^2(\frac{x}{2}+\frac{\pi}{12})$

Biến đổi phương trình đã cho tương đương với

$$\cos 2x-\sqrt{3}\sin 2x+10\cos(x+\frac{\pi}{6})+6=0$$

$$\Leftrightarrow \cos(2x+\frac{\pi}{3})+5\cos(x+\frac{\pi}{6})+3=0$$

$$\Leftrightarrow 2\cos^2(x+\frac{\pi}{6})+5\cos(x+\frac{\pi}{6})+2=0$$

Giải được $\cos(x+\frac{\pi}{6})=-\frac{1}{2}$ và $\cos(x+\frac{\pi}{6})=-2$ (loại)

*Giải $\cos(x+\frac{\pi}{6})=-\frac{1}{2}$ được nghiệm $x=\frac{\pi}{2}+k2\pi$ và $x=-\frac{5\pi}{6}+k2\pi$

58) (1 – tanx) (1+ sin2x) = 1 + tanx.

TXĐ: $x\neq\frac{\pi}{2}+l\pi \quad (l\in Z)$

Đặt t= tanx => $\sin 2x=\frac{2t}{1+t^2}$, đc pt: $(1-t)\left(1+\frac{2t}{1+t^2}\right)=1+t\Leftrightarrow \left[\begin{array}{l}t=0\\ t=-1\end{array}\right.$

Với $t=0$ => $x=k\pi,\ (k\in Z)$ (thoả mãn TXĐ)

Với t = -1 => $x=-\frac{\pi}{4}+k\pi$ (thoả mãn TXĐ)

59) $2\sin^2\left(x-\frac{\pi}{4}\right)=2\sin^2 x-\tan x$

Đk: $\cos x\neq 0(*)\ 2\sin^2\left(x-\frac{\pi}{4}\right)=2\sin^2 x-\tan x\Leftrightarrow 1-\cos\left(2x-\frac{\pi}{2}\right)=2\sin^2 x-\frac{\sin x}{\cos x}$

$\Leftrightarrow \cos x-\sin 2x.\cos x-2\sin^2 x.\cos x+\sin x\Leftrightarrow \cos x+\sin x-\sin 2x\left(\cos x+\sin x\right)=0$

$$\Leftrightarrow \left[\begin{array}{l}\sin x=-\cos x\xrightarrow{\cos x\neq 0}\tan x=-1\Leftrightarrow x=-\frac{\pi}{4}+k\pi\\ \sin 2x=1\Leftrightarrow 2x=\frac{\pi}{2}+l2\pi\Leftrightarrow x=\frac{\pi}{4}+l\pi\end{array}\right.\rightarrow x=\frac{\pi}{4}+k\frac{\pi}{2}$$

60) $\frac{\sin 2x}{\cos x}+\frac{\cos 2x}{\sin x}=tgx-\cot x$

$$(1)\Leftrightarrow \frac{\cos 2x\cos x+\sin 2x\sin x}{\sin x\cos x}=\frac{\sin x}{\cos x}-\frac{\cos x}{\sin x}$$

$$\Leftrightarrow \frac{\cos(2x-x)}{\sin x\cos x}=\frac{\sin^2 x-\cos^2 x}{\sin x\cos x}$$

$$\Leftrightarrow \cos x=-\cos 2x\ \wedge\ \sin 2x\neq 0$$

$$\Leftrightarrow 2\cos^2 x+\cos x-1=0\wedge \sin 2x\neq 0$$

$\Leftrightarrow \cos x = \frac{1}{2}$ $(\cos x = -1$:loại vì $\sin x \neq 0)$

$\Leftrightarrow x = \pm\frac{\pi}{3} + k2\pi$

61) $\frac{1}{\tan x + \cot 2x} = \frac{\sqrt{2}(\cos x - \sin x)}{\cot x - 1}$

Điều kiện:sinx.cosx $\neq 0$ và cotx $\neq 1$
Phơng trình tơng đơng

$$\frac{1}{\frac{\sin x}{\cos x} + \frac{\cos 2x}{\sin 2x}} = \frac{\sqrt{2}(\cos x - \sin x)}{\frac{\cos x}{\sin x} - 1}$$

$\Rightarrow \cos x = \frac{\sqrt{2}}{2} \Rightarrow x = \pm\frac{\pi}{4} + k2\pi$

Đối chiếu điều kiện pt có 1 họ nghiệm $x = -\frac{\pi}{4} + k2\pi$

62) $2\sqrt{2}\cos\left(\frac{5\pi}{12} - x\right)\sin x = 1$

$$2\sqrt{2}\cos\left(\frac{5\pi}{12} - x\right)\sin x = 1 \Leftrightarrow \sqrt{2}\left[\sin\left(2x - \frac{5\pi}{12}\right) + \sin\frac{5\pi}{12}\right] = 1$$

$$\Leftrightarrow \sin\left(2x - \frac{5\pi}{12}\right) + \sin\frac{5\pi}{12} = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4} \Leftrightarrow \sin\left(2x - \frac{5\pi}{12}\right) = \sin\frac{\pi}{4} - \sin\frac{5\pi}{12} =$$

$$= 2\cos\frac{\pi}{3}\sin\left(-\frac{\pi}{12}\right) = \sin\left(-\frac{\pi}{12}\right)$$

$$\Leftrightarrow \sin\left(2x - \frac{5\pi}{12}\right) = \sin\left(-\frac{\pi}{12}\right) \Leftrightarrow \left[\begin{array}{l} 2x - \frac{5\pi}{12} = -\frac{\pi}{12} + k2\pi \\ 2x - \frac{5\pi}{12} = \frac{13\pi}{12} + k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{6} + k\pi \\ x = \frac{3\pi}{4} + k\pi \end{array}\right. (k \in \mathbb{Z})$$